Cavity Perturbation in 1D waveguide cavity

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Maxwell equation: vector potential $\vec{A}$ and $\vec{F}$

$$\vec{H}_A = \frac{1}{\mu}\nabla \times \vec{A}$$

$$\vec{E}_F = -\frac{1}{\epsilon}\nabla \times \vec{F}$$

$$\nabla^2 \vec{A} + \beta^2 \vec{A} = -\mu \vec{J}$$

$$\nabla^2 \vec{F} + \beta^2 \vec{F} = -\epsilon \vec{M}$$

$$\vec{H}_A = \frac{1}{\mu}\nabla \times \vec{A}$$

$$\vec{E}_A = -j\omega\vec{A} - \frac{1}{\omega\epsilon\mu}\nabla(\nabla \cdot \vec{A})$$

$$\vec{E}_F = -\frac{1}{\epsilon}\nabla \times \vec{F}$$

$$\vec{H}_F = -j\omega\vec{F} - \frac{1}{\omega\epsilon\mu}\nabla(\nabla \cdot \vec{F})$$

$$\vec{E} = \vec{E}_A + \vec{E}_F$$

$$\vec{H} = \vec{H}_A + \vec{H}_F$$

TE$_{mn}$ modes

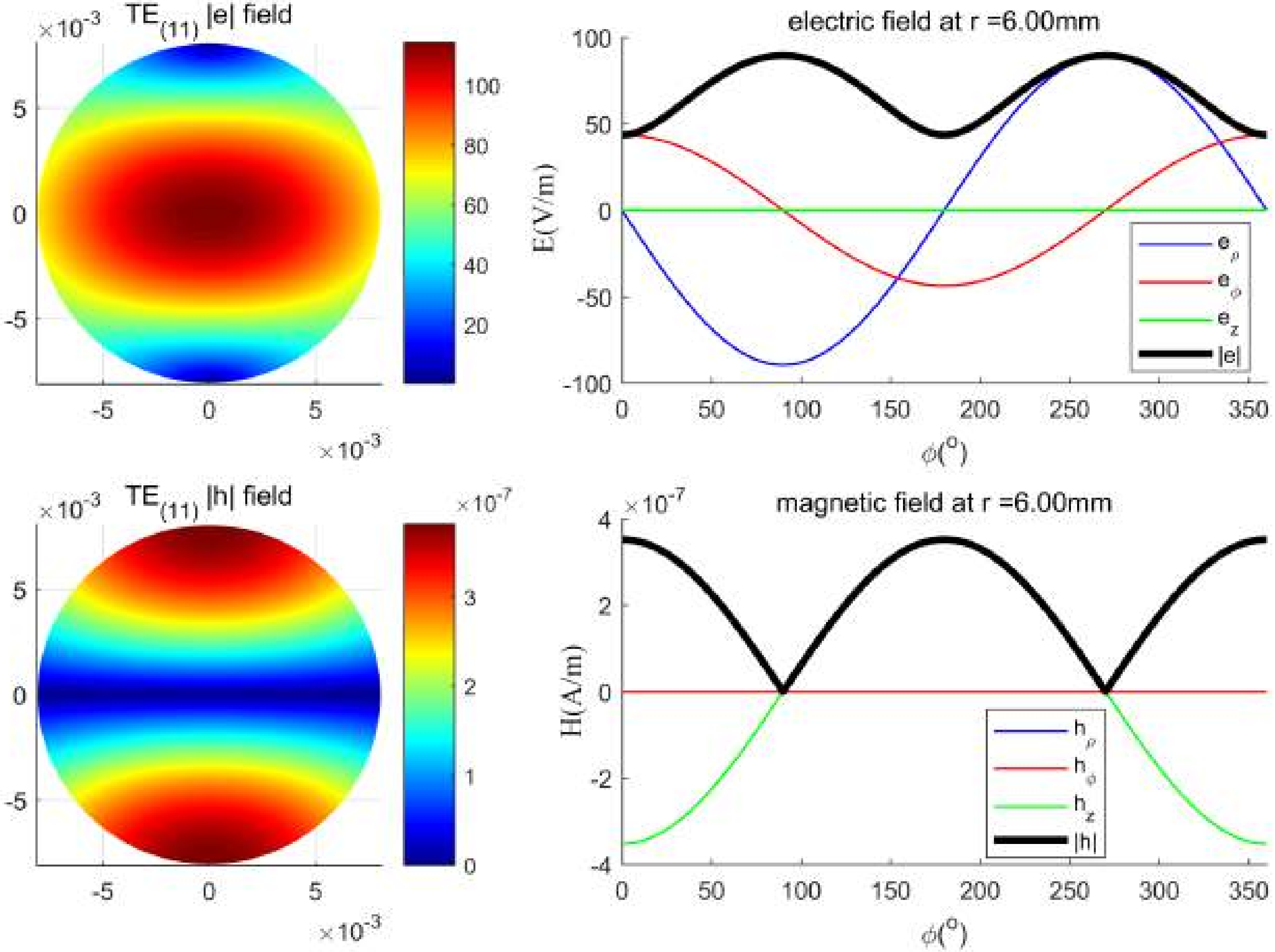
$$\vec{A} = 0$$
$$\vec{F} = F_z(\rho, \phi, z)\hat{z}$$
$$\nabla^2 \vec{F} + \beta^2 \vec{F} = 0$$

Boundary condition

$$E_\phi(\rho = a, \phi, z) = 0$$
$$E(\rho, \phi, z) < \text{infinite}$$
$$E(\rho, \phi, z) = E(\rho, \phi + 2\pi, z)$$

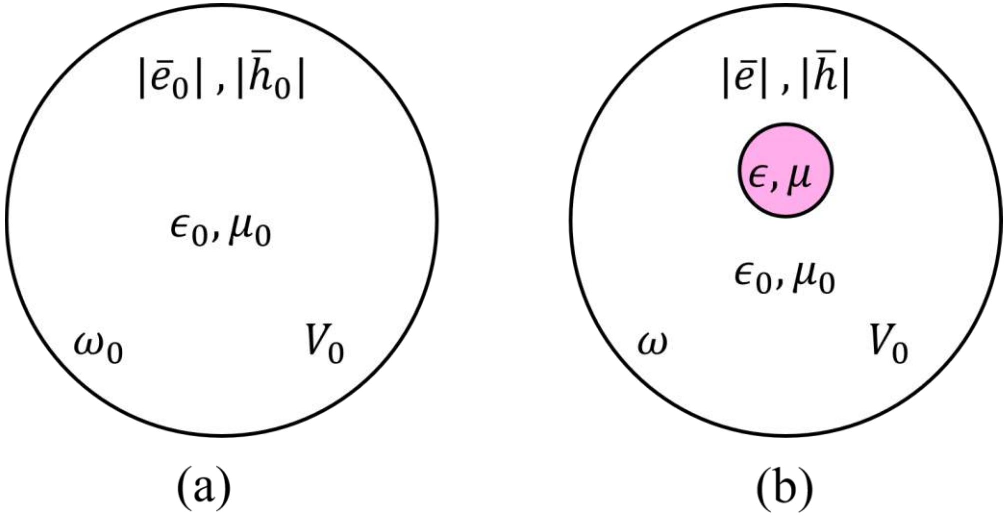
$$E_\rho = -A'_{mn}\frac{m}{\epsilon\rho}J_m(\beta_\rho\rho)cos(m\phi + \theta_c)e^{-i\beta_z z}$$
$$E_\phi = A'_{mn}\frac{\beta_\rho}{\epsilon}J'_m(\beta_\rho\rho)sin(m\phi + \theta_c)e^{-i\beta_z z}$$
$$E_z = 0$$
$$H_\rho = -A'_{mn}\frac{\beta_\rho\beta_z}{\omega\epsilon\mu}J'_m(\beta_\rho\rho)sin(m\phi + \theta_c)e^{-i\beta_z z}$$
$$H_\phi = -A'_{mn}\frac{m\beta_z}{\omega\epsilon\mu\rho}J_m(\beta_\rho\rho)cos(m\phi + \theta_c)e^{-i\beta_z z}$$
$$H_z = -iA'_{mn}\frac{\beta\rho^2}{\omega\epsilon\mu}J_m(\beta_\rho\rho)sin(m\phi + \theta_c)e^{-i\beta_z z}$$

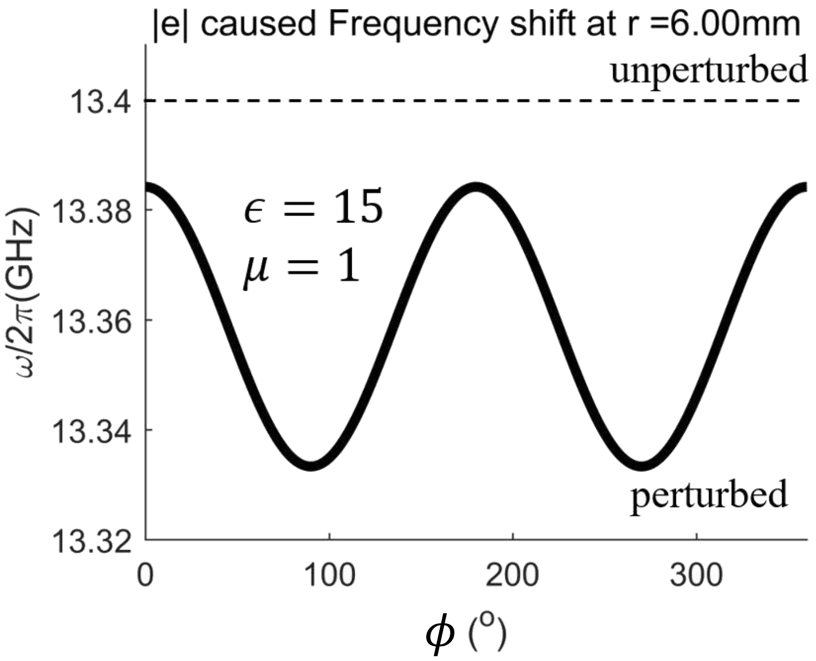
TE_{11} modes

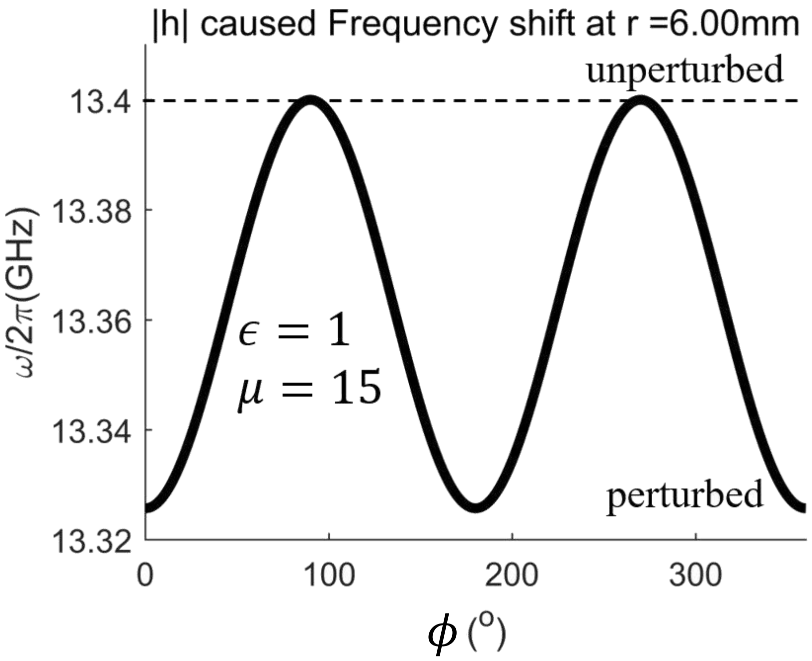

TE(11) |e| field
electric field at r =6.00mm
E(V/m)
φ(°)
e_ρ
e_φ
e_z
|e|
TE(11) |h| field
magnetic field at r =6.00mm
H(A/m)
h_ρ
h_φ
h_z
|h|
×10^-3
×10^-7

Microwave perturbation theory

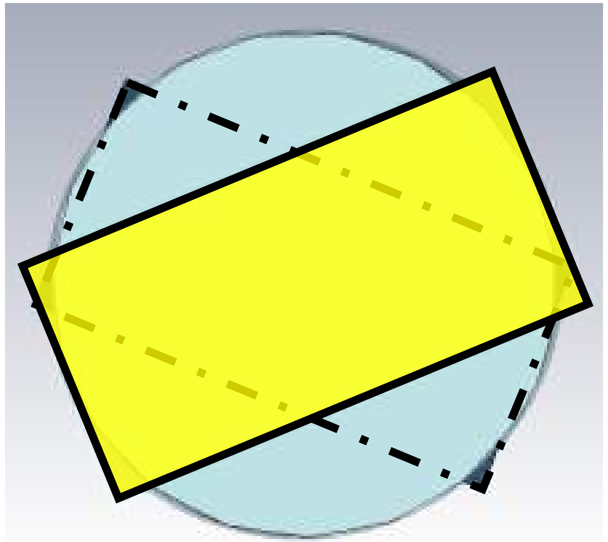
$$\frac{\omega - \omega_0}{\omega_0} \approx -\frac{\iiint_v [\ \Delta\mu |H_0|^2| + \Delta\epsilon |E_0|^2\ ]dV}{\iiint_v [\ \mu_0 |H_0| + \epsilon_0 |E_0|^2\ ]dV}$$



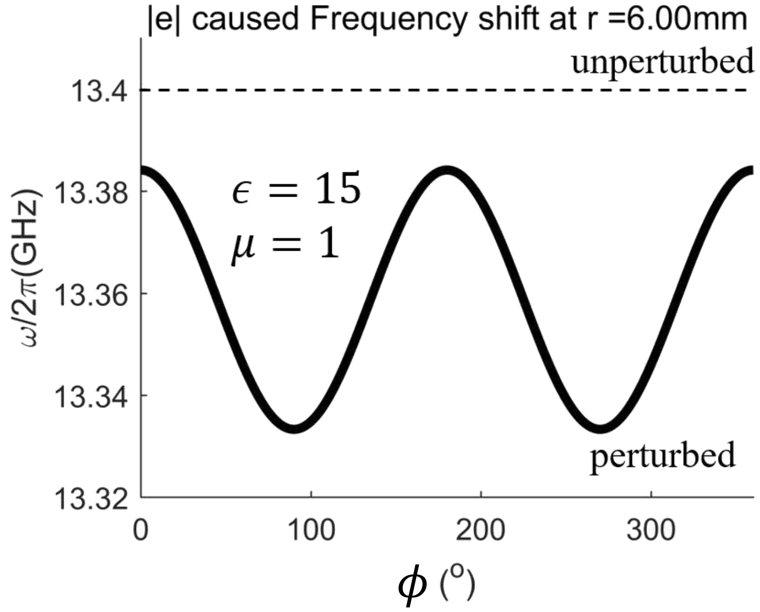


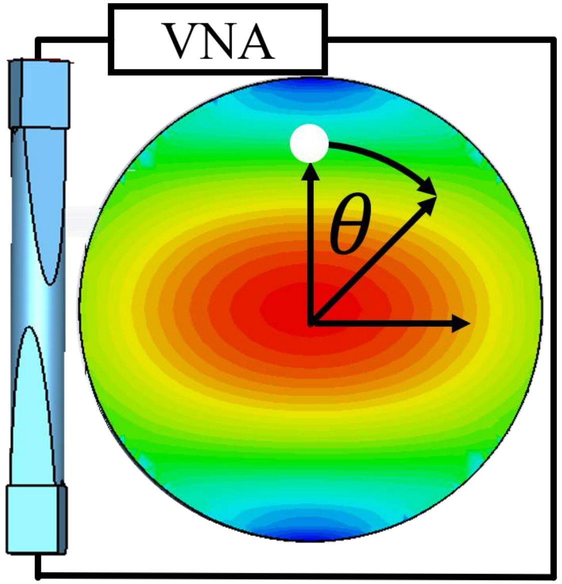


Experiment

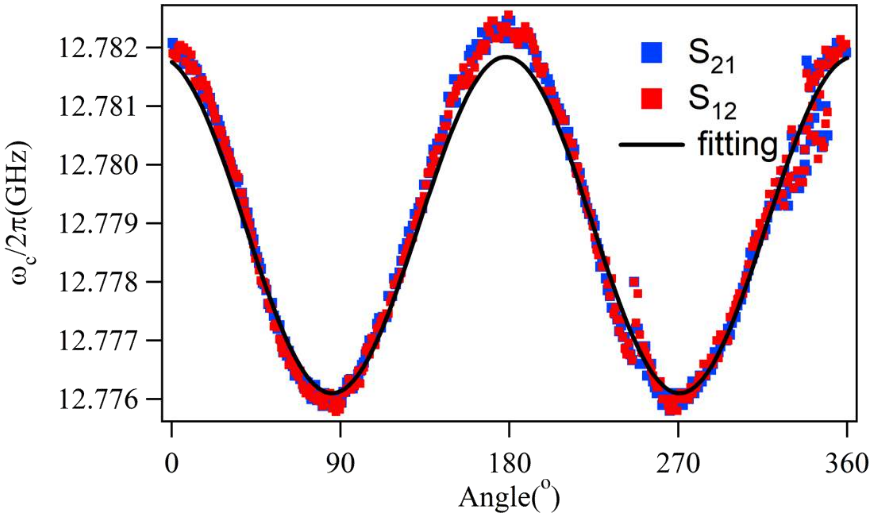


Front view



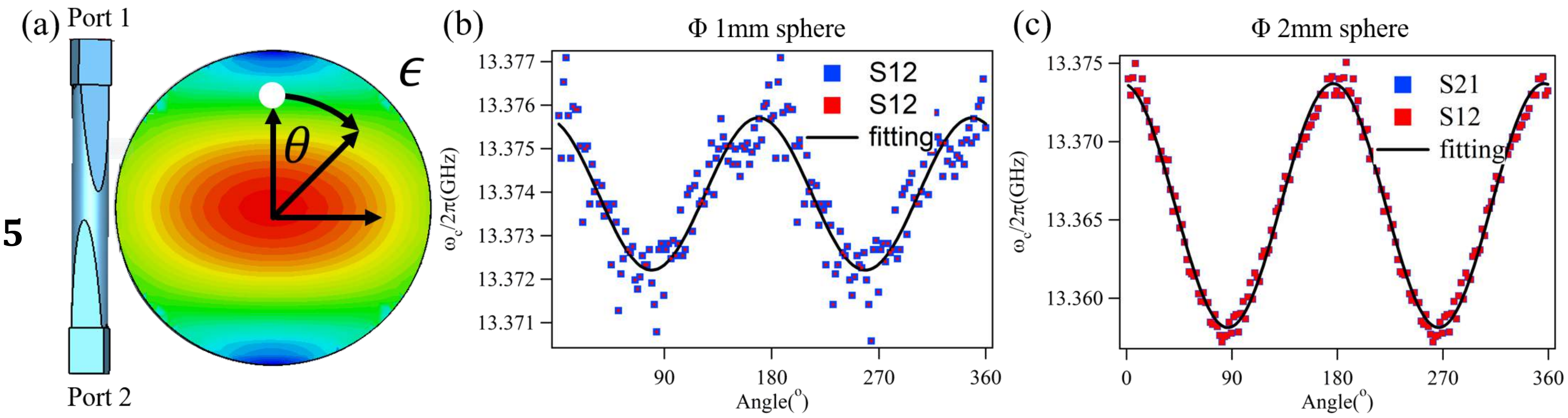


Top view

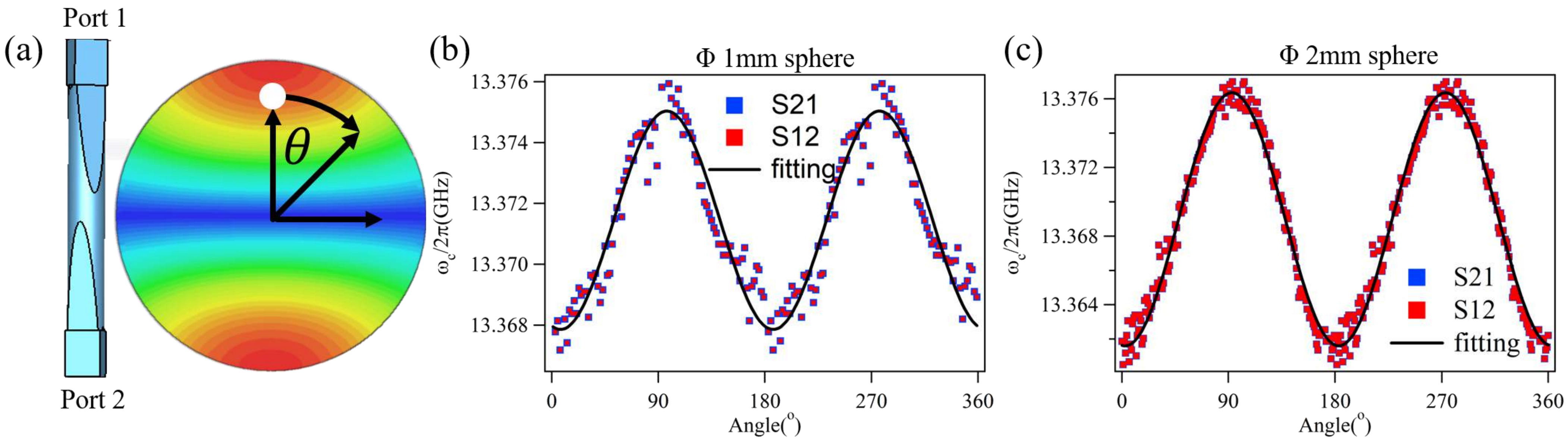


Simulations:

$\epsilon = 1$
$\mu = 15$



$\epsilon = 15$
$\mu = 1$



Next step:

1. Derive the coupling strength g as a function of field distribution, and why $g^2 < 0$.

2. Explain why the largest shift (minimum h distribution) is at angle bisector.